H/w 11·1

Qu 10

$y = \cos \omega t$

$y' = -\omega \sin \omega t$ and $y'' = -\omega^2 \cos \omega t$.

substituting this in

$$\frac{d^2 y}{dt^2} + 9y = 0$$

we have

$$-\omega^2 \cos \omega t + 9 \cos \omega t = 0$$

$$(9 - \omega^2) \cos \omega t = 0$$

&

$$9 - \omega^2 = 0 \Rightarrow \omega^2 = 9 \quad \& \quad \omega = \pm 3$$ //

Comment.

Many of you forgot that the square root of 9 is +3 and -3.

H/w 11.2

Q48.

a, $\overline{\underline{II}}$; because the slope of is negative above the x-axis when (y is positive) and positive below the x-axis (when y negative)

b, $\overline{\underline{I}}$; the slope is positive for positive y values and negative for negative y.
As y gets larger the slope gets larger.

c, $\overline{\underline{V}}$; slope is positive for x being positive and negative for negative x.

d, $\overline{\underline{III}}$; slope is positive for positive y and negative for negative y,
As y ~~approaches~~ approaches 0 the slope gets large

e, $\overline{\underline{IV}}$; slope will always be positive

H/w 11.3

Q4

Using Euler's method to find an approximation for $y(2)$

we repeat for $\Delta x = 0.2,\ 0.1,\ 0.05$.

eg.

$\Delta x = 0.2$, initial value $y(1) = 1 \Rightarrow P_0(1, 1)$

slope $= \sin(xy) = \sin 1$ (in radians)

~~$\Delta y = \Delta x \sin(1 \cdot 1)$~~

~~$= 0.2\ (0.1683$~~

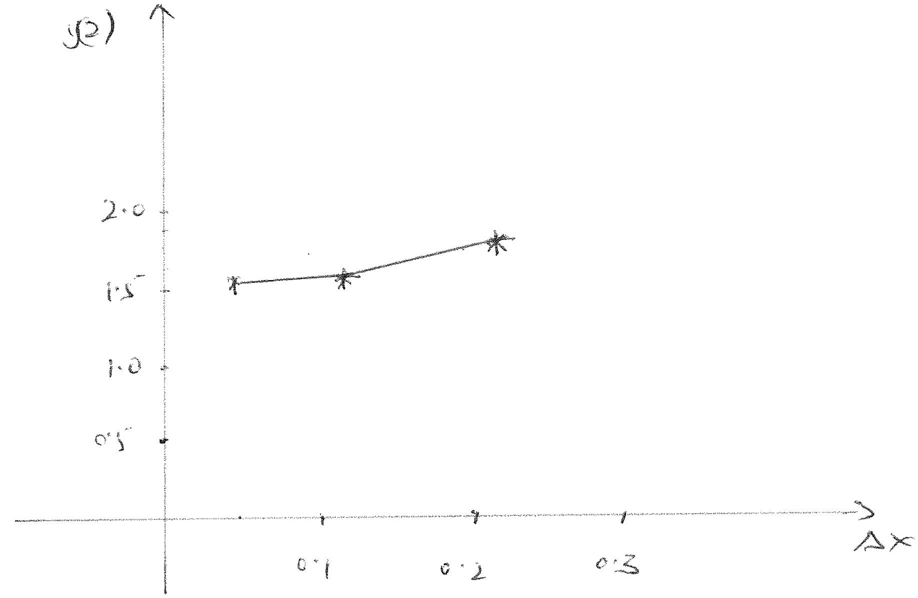
$y(1.2) \approx y(1) + \Delta x \sin(1 \cdot y(1)) = 1.1683.$

$(1.2, 1.1683)$.

Computed solutions to 4 decimal places

x	for $\Delta x = 0.2$	$\Delta x = 0.1$	$\Delta x = 0.05$
1.0	1	1	1
1.1		1.0841	1.0865
1.2	1.1683	1.1771	1.1812
1.3		1.2758	1.2806
1.4	1.3655	1.3754	1.3791
1.5		1.4692	1.4699
1.6	1.5539	1.5498	1.5461
1.7		1.6113	1.6027
1.8	1.6758	1.6505	1.6378
2.0	1.7008	1.6648	1.6482

a plot of $y(2)$ against Δx will be a straight line (approximately)



when Δx approaches zero the ~~straight line~~ plots of $y(2)$ converge to the exact value of $y(2)$

this is the y intercept hence $y(2) \approx 1.632$ //

Comments

* Most of you forgot the additional questions asked, plotting $y(2)/\Delta x$ concluding from the graph and finding the exact value for $y(2)$.

In calculating the slope of the differential equation give

$dy/dx = \sin xy$

a few of you solved for the slope in degrees instead of radians. ie slope $= \sin(1 \cdot y(1))$.

H/W 11.4

Qu 32

for $\frac{dQ}{dt} = b - Q$

we take the integral on both side:

$$\int \frac{dQ}{b-Q} = \int dt$$

$$-\ln|b-Q| = t + c$$

$$b - Q = e^{-(t+c)} = e^{-t}e^{-c}$$

then

$$Q = b - e^{-t}e^{-c}$$

$$= b - Ae^{-t}.$$

where $A = \pm e^{-c}$ or $A = 0$.

Comments.

A major problem here ~~we~~ for most of you was the algebra of the negative sign.

especially $-\ln|b-Q| = t+c \Rightarrow b-Q = e^{-(t+c)}$